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Dynamical Systems Modeling to Identify a Cohort of Problem Drinkers with Similar Mechanisms of Behavior Change

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Abstract

One challenge to understanding mechanisms of behavior change (MOBC) completely among individuals with alcohol use disorder is that processes of change are theorized to be complex, dynamic (time varying), and at times non-linear, and they interact with each other to influence alcohol consumption. We used dynamical systems modeling to better understand MOBC within a cohort of problem drinkers undergoing treatment. We fit a mathematical model to ecological momentary assessment data from individual patients who successfully reduced their drinking by the end of the treatment. The model solutions agreed with the trend of the data reasonably well, suggesting the cohort patients have similar MOBC. This work demonstrates using a personalized approach to psychological research, which complements standard statistical approaches that are often applied at the population level.

Keywords: Mathematical psychology, inverse problems, behavior change, personalized medicine, dynamical modeling, ecological momentary assessment data.

Introduction

Recently, the National Institute of Health proposed precision medicine as a means to improve the efficiency and effectiveness of treatments of all disease ("Precision Medicine," 2016). The primary principle of precision medicine is that one aims to identify the unique components of both health and disease of each individual so that an extremely tailored and targeted intervention or set of interventions can be provided to the individual to maximize their efficacy. Where previously medicine pursued a one size fits all treatment, or the treatment that was most effective for the most people, emphases have now been placed on an individually tailored approach in order to advance healthcare to its next generation. In taking such a perspective, the focus of research must shift to include how individuals differ from group averages.

One disease to which precision medicine can be applied is alcohol use disorder (AUD). Excessive alcohol consumption is known to cause the deaths of about 88,000 people each year in the United States and is associated with an estimated public health cost of about $249 billion in

2010 (“Alcohol Use And Your Health,” 2016). While a number of treatment interventions are available for AUD, treatments remain only modestly effective (Longabaugh, 2013). In order to improve interventions for individuals with AUD, mechanisms of behavior change (MOBC) both for treatment related and self-initiated drinking reduction need to be understood (Huebner & Tonigan, 2007). Identifying MOBC can help healthcare providers implement more efficient and effective interventions by understanding the crucial factors that initiate and maintain the change process.

A critical challenge to obtaining a more complete understanding of MOBC among individuals with AUD is that processes of change are theorized to be complex, dynamic (time varying), and at times non-linear, all of which may interact with each other to influence alcohol consumption. Such complexity presents challenges to both data collection and data analyses. One way to better understand these change processes is by collecting data on individuals as they interact with their natural environment in real time. Extensive, real time data collection can occur inexpensively, efficiently, and accurately using ecological momentary assessment (EMA), which is designed to collect ecologically valid data about behavior, thoughts, and feelings over time, while avoiding the pitfalls of retrospective recall (Shiffman, 2009).

In conjunction with EMA, mathematical modeling can be utilized to understand these complex, highly interactive, time varying, and non-linear data. While advanced statistical procedures can be used effectively with intensive, longitudinal datasets (e.g., Boker & Laurenceau, 2006), such statistical procedures tend to reduce results to averages across individuals, thereby limiting the amount of information that might be gleaned from a particular dataset. Mathematical modeling provides an exciting compliment to such methods by modeling time varying relationships between variables and nonlinear systems represented by repeated measurement data (Davidian & Giltinan, 1995).

Mathematical modeling has already been used as a method of understanding social behaviors. Since cyclic patterns are a fundamental element of many psychological theories (Chow et al., 2009), mathematical oscillator models have been utilized to help improve the understanding of these processes. For example, oscillation models have been used to describe the dynamics of several psychological constructs, such as emotion, stress and affect, and intimacy (Bisconti et al., 2004; Boker & Laurenceau, 2006; Chow et al., 2005; Montpetit., 2010). Mathematical modeling efforts in the context of alcohol consumption have been mainly implemented at the population level. For example, previous efforts applied mathematical epidemiology techniques to reflect alcohol-related behavior in populations (Sanchez et al., 2007).

Previously, a dynamical systems modeling approach was initiated to understand the changes in drinking behaviors at a personal level (Banks et al., 2014). In this study, the authors investigated several key factors related to MOBC among individuals with AUD. In a subsequent study (Banks et al., 2016), the authors then applied this new approach to build a preliminary model of behavior change. They relied on theories of behavior change related to substance abuse in developing the model and selecting four primary variables that vary over time.

In the present work, we extended this modeling effort. We first identified a cohort of participants from a sample of problem drinkers recruited into a randomized controlled trial of brief treatment for AUD called Project SMART (Morgenstern et al., 2012). The participants selected for this cohort successfully reduced their drinking during treatment and were hypothesized to share the same underlying MOBC in alcohol consumption. We then developed and honed a mathematical model using each of their data during the iterative process of modeling to determine the relationships between the identified variables.

Method

Project SMART was a study that tested the combined effectiveness of modified behavioral self-control therapy (MBSCT) and naltrexone (NTX) in problem drinking men who have sex with men (MSM) (Morgenstern et al., 2012).

Participants

Participants responded to online and print advertisements targeting MSM who wished to reduce but not stop drinking. To be eligible for this study, men had to: be drinking greater than 24 standard drinks per week; identify as sexually active with other men over the preceding 90 days; and read English at an eighth-grade level or higher. Participants were excluded if they: 1) had a lifetime diagnosis of bipolar disorder, schizophrenia, or other psychotic disorders; 2) an untreated current major depressive disorder; or 3) current physiological dependence on alcohol or other drugs (with the exception of nicotine or cannabis), as demonstrated by current physical withdrawal symptoms or a history of severe withdrawal syndrome; 4) started or changed psychotropic medication in the preceding 90 days; 5) were at risk for serious medication side effects from naltrexone; 6) reported regular use of opioids; or 7) were enrolled in concurrent drug- or alcohol-related treatment during the 12-week treatment phase of the study (Morgenstern et al., 2012). The typical participant was male, approximately 40 years old, Caucasian, attended at least some college, and employed.

Procedures

After initial screening, eligible and enrolled participants (N = 200) were randomized to one of four conditions: placebo only (PBO), naltrexone only (NTX), Modified Behavioral Self-Control Therapy only (MBSCT), or both naltrexone and MBSCT (NTX + MBSCT). At the end of 12 weeks of treatment, all participants received a follow-up assessment.

Study Interventions. All participants received Brief Behavioral Compliance Enhancement Treatment (BBCET), a series of 20-minute sessions with a psychiatrist weekly for the first three weeks, and then every other week thereafter. Participants were blind to medication condition. Dosage of NTX was initiated at 25 mg/day, and then increased to 100 mg/day during the first three weeks of treatment. For those who received MBSCT, treatment was a combination of motivational interviewing and cognitive behavioral therapy and comprised of 12 one-hour psychotherapy sessions that focused on moderation as a goal.

Ecological Momentary Assessment: Daily Diary. All participants completed a daily telephone survey delivered via Interactive Voice Recording (IVR) (TELESAGE, 2005) between 4:00 pm and 10:00 pm each day, for a total of 84 days. The questionnaire consisted of 30-45 questions and collected information related to emotions, daily events, and drinking behaviors. Participants received an automated reminder call if they failed to call into the system by 8:00 pm. Each survey required between 2 to 5 minutes to complete.

Measures

All four measures used in this study were from EMA.

Daily alcohol consumption. Alcohol consumption was assessed by having participants report the number of standard drinks of beer, wine, and liquor consumed in the past 24 hours.

Norm violation. Norm Violation was assessed by asking, *Do you consider the total amount you have had to drink since this time yesterday to be excessive? That is, was it more than you think you should have had?* The response set ranged from 0 (*Definitely Not*) to 3 (*Definitely*).

Personal norm. The thresholds (i.e., norms) individuals used to evaluate whether or not their drinking was excessive is referred to here as "personal norm". Personal norms vary across individuals and can be considered to be dynamic across time and setting of drinking. Personal norm in this study is a latent variable and thus was not directly measured. We include this latent variable in our modeling process.

Confidence. Confidence was measured by asking, *how confident are you that you can resist drinking heavily (that is, resist drinking more than 4 standard drinks) over the next 24 hours?* The response set ranged from 0 (*Not at all*) to 4 (*Extremely*).

Commitment. Commitment was measured by asking, *How committed are you not to drink heavily (that is, not to drink more than 4 standard drinks) over the next 24 hours?* The response set ranged from 0 (*Not at all*) to 4 (*Extremely).*

Analytic Plan

Variable selection. Based on findings from previous studies (Kuerbis et al., 2014; Morgenstern et al., 2016), a dual process theoretical framework for substance abuse (Morgenstern et al., 2013) was utilized 1) to select four key variables that directly relate to the number of drinks consumed, and 2) to try to understand how those variables interact with each other over time. The dual process framework for addiction proposes a top-down, bottom-up cognitive process in which top-down executive functioning (e.g., commitment not to drink) attempts to control responses to stimuli (e.g., alcohol) which also evoke implicit cognitive processes. The variables identified to represent the dual process model were: alcohol consumption, norm violation, confidence, and commitment. While desire was included as a constant factor in the model, it was excluded as a variable of focus from this initial iterative model building process. Further exploration of desire will occur during a future stage of model development.

Mathematical modeling methodology and participant selection. Mathematical models can represent and describe psychological processes using mathematical expressions. The dynamical modeling approach used here to examine MOBC is an iterative process (Figure 1). In general, a preliminary mathematical model is proposed based on existing psychological theories and empirical observations. Then results are compared to the observed data to evaluate how accurately the model describes the underlying psychological process. This evaluation should either confirm existing psychological theories or lead to a new psychological understanding of the relationships among the variables. The latter can then lead to model adjustment and a repeated cycle. The mathematical model quantifies how the key variables change over time and how they interact among each other.

The psychological process described by the mathematical model depends on parameters, which are often unknown or not directly measurable. These unknown parameters are often estimated by solving an *inverse problem*, which is, given an individual's dataset and mathematical model, the problem of estimating parameters that would generate such a dataset. The resulting parameters should minimize the distance between the model solution and the data. The model solution is personalized for that individual according to the particular set of parameters.

Before solving an inverse problem, the correct statistical error model needs to be identified in order to account for the uncertainty in the data (observation error). Mis-specifying the error structure can lead to an incorrect estimation of the parameters (Banks, et al., 2014; Banks & Tran, 2009). If the error does not depend on the size of the observations (i.e., the error is evenly distributed across various observation sizes), an ordinary least squares method is appropriate for parameter estimation; if, however, the error depends on the size of the observation (i.e., the error does not remain constant over observation sizes), an Iterative Weighted Least Squares (IWLS) method is required.

To account for the uncertainty in the data, let $Y_{i,j}$ be a random variable associated with collected data for mathematical model variable i at time j. Consider the following statistical error model

$$Y_{1,j} = f_1(t_j;\boldsymbol{\theta}_0) + f_1(t_j;\boldsymbol{\theta}_0)^{\gamma_1}\mathcal{E}_{1,j}$$
$$\vdots$$
$$Y_{4,j} = f_4(t_j;\boldsymbol{\theta}_0) + f_4(t_j;\boldsymbol{\theta}_0)^{\gamma_4}\mathcal{E}_{4,j},$$

where $f_1(t_j;\boldsymbol{\Theta}_0),\ldots, f_4(t_j;\boldsymbol{\Theta}_0)$ represent the mathematical model solution for variables alcohol consumption, norm violation, confidence, and commitment, respectively (see the mathematical model below) at time j with the nominal parameter vector $\boldsymbol{\Theta}_0$, which is assumed to exist. The term $f_i(t_j;\boldsymbol{\theta}_0)^{\gamma_i}\mathcal{E}_{i,j}$ represents the measurement error that causes the data to not exactly equal $f_i(t_j;\boldsymbol{\theta}_0)$. We assume the random vector[1] $\boldsymbol{\mathcal{E}}_j = (\mathcal{E}_{1,j},\ldots,\mathcal{E}_{4,j})^T$ are independent and identically distributed (i.i.d) with mean zero. We represent the obtained data, $y_{i,j}$, collected at time j for variable i, for $j = 1,\ldots,n$ by the following

$$y_{1,j} = f_1(t_j;\boldsymbol{\theta}_0) + f_1(t_j;\boldsymbol{\theta}_0)^{\gamma_1}\epsilon_{1,j}$$
$$\vdots$$
$$y_{4,j} = f_4(t_j;\boldsymbol{\theta}_0) + f_4(t_j;\boldsymbol{\theta}_0)^{\gamma_4}\epsilon_{4,j}$$

where $\epsilon_{i,j}$ is a realization of the random variable $\mathcal{E}_{i,j}$. See Banks et al. (2016) for further details on the statistical error model and implementation of the IWLS method.

In Banks et al. (2016) the mathematical model was developed using one participant's data. In this study, we continue the iterative modeling process by both slightly improving the mathematical model and by applying this model to three additional patients who reduced their drinking. These three patients were selected as they had more complete data and were considered to be "treatment responders" by visually determining a dramatic reduction in their drinking over the treatment period. (We further address the selection of these patients in the study limitation section.) We fit the mathematical model to each of them and determined they shared a common set of mechanisms. These patients were then identified as a cohort.

Based on the psychological hypothesis presented in Banks et al. (2016), we formulated the mathematical model. For each patient's dataset, we then determined the correct statistical error model using a *second-order differencing method* to quantify the observation error for alcohol consumption, norm violation, confidence, and commitment (Banks et al., 2016). The results revealed that the IWLS method was appropriate in our case with $\boldsymbol{\gamma} = [\gamma_1,\gamma_2,\gamma_3,\gamma_4]$, where γ_1, γ_2, γ_3 and γ_4 correspond to alcohol consumption, norm violation, confidence, and commitment respecttively for each individual patient. We finally solved the inverse problem to estimate the patient-specific mathematical model parameters and compare the model solution to each patient's dataset.

Mathematical Model

Here we present the mathematical model from Banks et al. (2016) with a modification that better quantifies the trend of commitment in the selected patients. A schematic (Figure 2) representing the relationships among the variables is created based on prior psychological knowledge and observations of the data.

Timing of variables

The variable $A(t)$ represents daily alcohol consumption, or the number of alcoholic drinks a person has consumed in the past 24 hours from time t (i.e., from timet t - 1 to time t). $V(t)$ represents norm violation on a particular day (time t). Norm violation also relates to the period between t – 1 and t. $C_f(t)$ represents the confidence level a person feels at time t that he can resist drinking heavily in the next 24 hours (i.e., from time t to time t + 1). $C_h(t)$ represents the commitment a person makes to not drink heavily in the next 24 hours at time t (i.e., from time t to time t + 1).

Schematic model to mathematical model

The model is built by formulating equations that represent the hypothesized relationships demonstrated in Figure 2. Each arrow in the schematic diagram corresponds to a term in the model (Banks et al, 2016). For example, arrow 2 in Figure 2 corresponds to term 2 in Equation (1a) such that if the participant feels that his drinking in the past 24 hours violated his personal norm, his drinking will decrease in the next 24 hours.

[1] T represents transpose of a vector

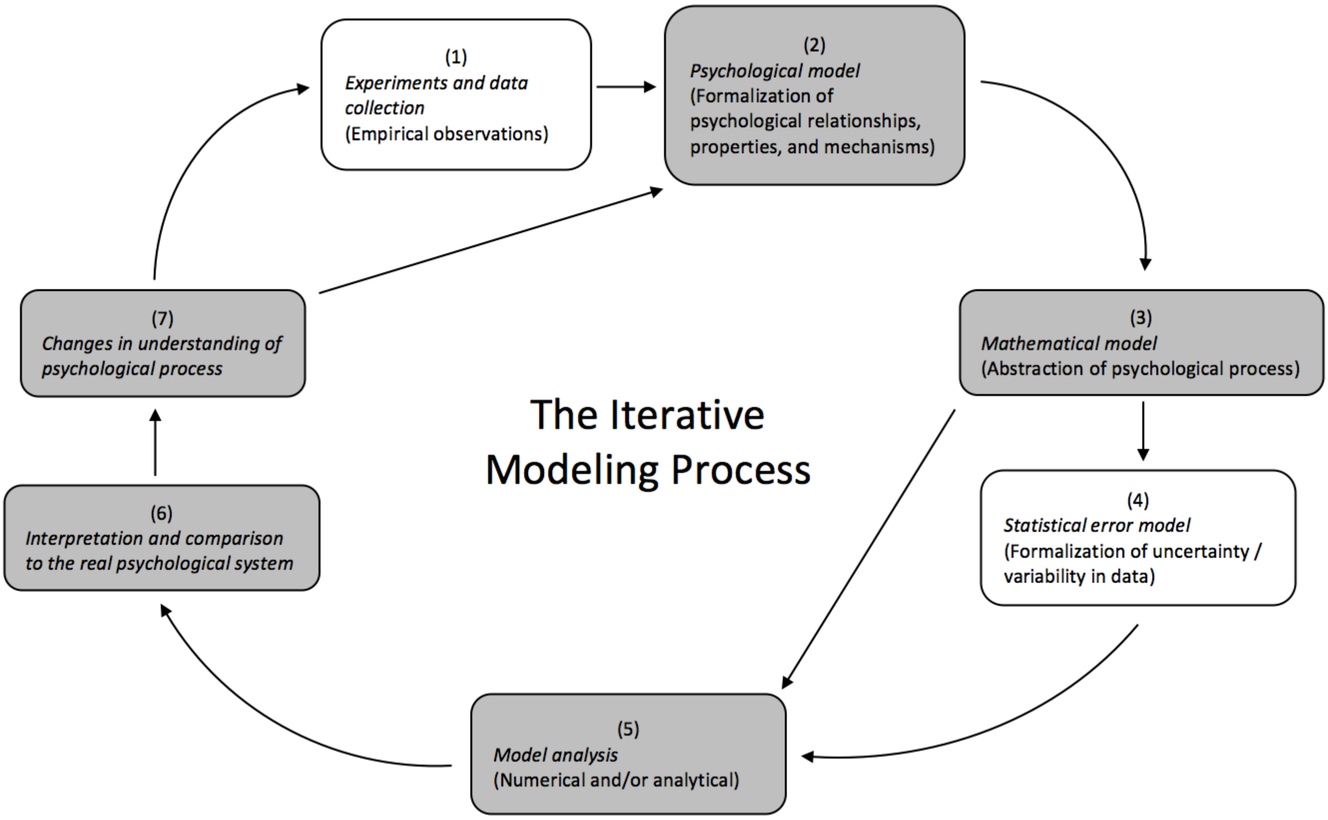


Figure 1. The Iterative Modeling Process (Banks & Tran, 2009). The white boxes indicate steps when data is available.

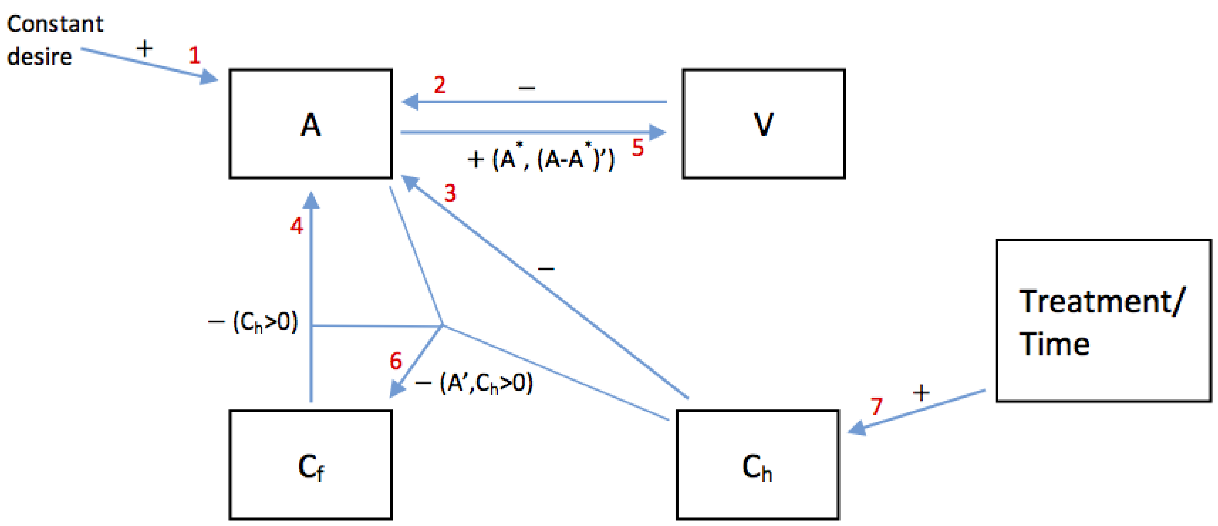


Figure 2. Schematic of hypothesized variable relationships (Banks et al., 2016). A represents alcohol consumption, A^* represents the number of drinks that a person believes to be his norm, V represents norm violation, C_f represents confidence, and C_h represents commitment. The arrows represent the hypothesized causal relationships between the variables.

The mathematical model is given by the following

$$\frac{dA}{dt} = \underbrace{a_1}_{1} - \underbrace{a_2 V(t-1)}_{2} - \underbrace{a_3 C_h(t-1)}_{3} - \underbrace{a_4 C_f(t-1) C_h(t-1)}_{4} \tag{1a}$$

$$\frac{dV}{dt} = \underbrace{\chi_{(A>A^*)} v_1 \frac{d(A-A^*)}{dt}}_{5a} - \underbrace{\chi_{(A\leq A^*)} v_2 V}_{5b} \tag{1b}$$

$$\frac{dC_f}{dt} = \underbrace{-\chi_{(A>5)} d_1 \frac{dA}{dt} C_h(t-1)}_{6a} + \chi_{(A\leq 5)} \underbrace{d_2 (C_f - \alpha)\left(1 - \frac{C_f}{n}\right)}_{6b} \tag{1c}$$

$$\frac{dC_h}{dt} = \underbrace{m C_h \left(1 - \frac{C_h}{K}\right)}_{7} \tag{1d}$$

where

$$A^*(t) = \underbrace{b e^{-rt} + l}_{8}, \tag{1e}$$

and χ is the indicator function defined as follows

$$\chi_{(A>x)} = \begin{cases} 1 \; if \; A > x \\ 0 \; else \end{cases}, \; \chi_{(A\leq x)} = \begin{cases} 1 \; if \; A \leq x \\ 0 \; else \end{cases}, \tag{1f}$$

where $x = A^*$ in (1b) and $x = 5$ in (1c).

The equations above include the hypothesized MOBC based on theories of behavior change and our previous studies. For this particular model, we assume desire to be constant.

Individual equations

In Equation (1a), term 1 describes how the rate of change of alcohol consumption is increased by one's desire to drink, which is held constant here. Terms 2 and 3 describe how the rate of drinking decreases if the participant considers his drinking in the past 24 hours to be excessive, and if he is committed to not drink heavily, respectively. In addition, if the participant feels both confident and committed that he can resist drinking heavily in the next 24 hours, then his alcohol consumption decreases. However, if the participant feels confident but definitely not committed, then his confidence level will not affect his alcohol consumption (term 4).

Equation (1b) describes how the rate of change in norm violation depends on the patient's alcohol consumption relative to his personal norm, A*. If the participant drank less than or equal to his personal norm in the past 24 hours, his norm violation will decrease exponentially to 0 (term 5b). If the participant drank more than his personal norm in the past 24 hours, the change in norm violation is dependent on the rate at which the number of drinks approaches the personal norm, denoted by $\frac{d(A-A^*)}{dt}$ (term 5a). His norm violation decreases if his alcohol consumption decreases towards his personal norm at faster rate than the rate of decrease in his personal norm. His norm violation increases if his alcohol consumption decreases at a slower rate compared to his personal norm or if his alcohol consumption increases away from his personal norm.

Equation (1c) describes that the rate of change in confidence depends on whether the patient is drinking heavily or not (recall that the National Institute on Alcohol Abuse and Alcoholism uses 5 drinks as the threshold for drinking heavily):

1. If the participant drinks heavily (more than 5 drinks in last 24 hours) and he feels committed, his confidence depends on the rate of alcohol

consumption, $\frac{dA}{dt}$; if the patient's alcohol consumption is increasing (decreasing) over time, then his confidence will decrease (increase).

2. In addition, if the participant drinks less than 5 drinks in 24 hours, then his confidence will increase logistically (Figure 3). The logistic model is well established and often used in biomathematics. For further information on this model, see (Banks, 1975; Kot, 2001). When the participant first stops drinking heavily ($R1$, bottom of the curve), he will need to establish a habit of drinking lightly for a few days. As he gains a sense of mastery, his confidence will increase more quickly towards his maximum confidence level ($R2$, steep, middle of the curve). After the participant has mastered this habit of drinking lightly, his increase in confidence slows as he "reaches" his maximum confidence level ($R3$, top of the curve).

Equation (1d) captures the hypothesis that a participant's motivation level (i.e., commitment) increases as the treatment period progresses. We quantify this increase using a logistic model rather than the previous function presented in (Banks et al., 2016) to allow for a slower increase in commitment at the beginning of treatment.

Equation (1e) describes that the personal norm decreases during the treatment period.

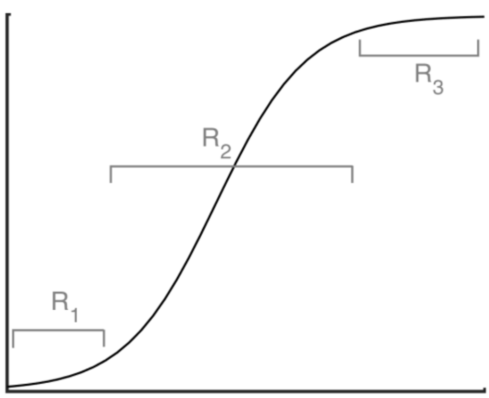


Figure 3: Logistic model (Banks et al., 2016)

Model Solutions Describing MOBC

Below we present the results for four participants (condition noted in parentheses), 1761 (MBSCT), 1771 (MBSCT), 1474 (NTX + MBSCT) and 1460 (NTX). As we can see in Figures 4 - 7, the model describes the relationships among the variables reasonably well. The data in the figures are averaged weekly IVR data. As it will be described below, we use these averaged data to better show the overall trend of the data; we use patient 1474 to illustrate that as we average over 3, 5, and 7 days, it becomes more obvious that the trend in the data is captured by the model. Similar results can be found for all four patients in the supplemental material section. We then discuss the results for each patient.

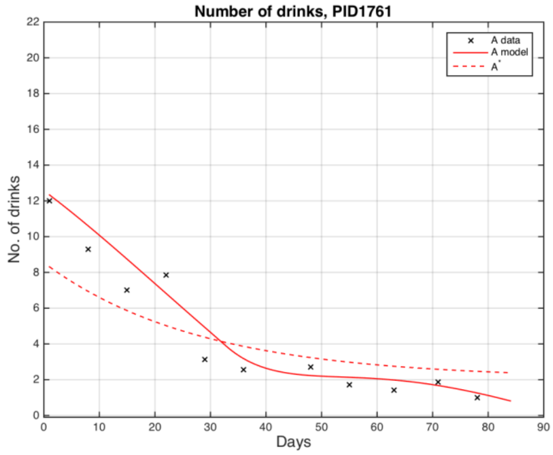
Rationale of Using Average vs. Daily Data

As mentioned above, the model solutions presented for the four patients are fit to the IVR data averaged weekly in order to better show the trend of the data over the course of the treatment period. Initially, we fit the model to the daily IVR data. However, we were interested in modeling the general trend of the data rather than the daily fluctuations. Due to the nature of the data (qualitative or Likert type data [Likert, 1932]), we found that it is difficult to determine if the continuous model solutions follow the dynamics in the data on a fine scale. Therefore, we averaged the data over 3, 5, and 7 days and fit the model to these modified datasets.

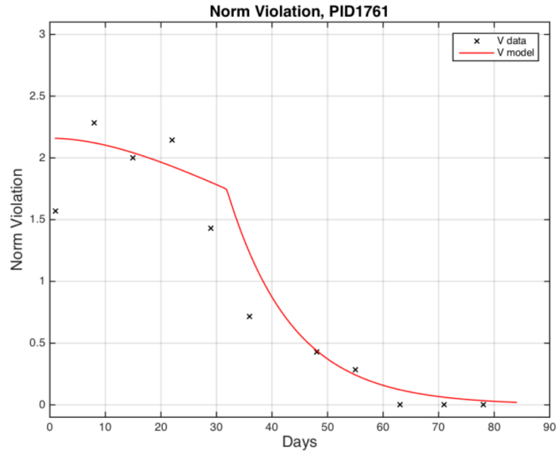
To illustrate how averaging can better show the overall trend in the data, we present the results for a sample patient (PID 1474). Figures 8 - 10 and 6 contain the original data, and data averaged over 3, 5, and 7 days, respectively for this sample patient. Note that 'o' represents the daily data while 'x' represents the averaged data. Each figure also contains the corresponding model solution for that dataset. Notice that as more data is averaged, the trend in the data and the agreement with model solutions becomes more apparent. For example, the data in Figure 8c looks scattered and it is not obvious that the model solution represents the overall confidence dynamics. Even though the model solution is visually similar for each dataset (Figures 9c, 10c, 6c), as the data is averaged over longer time periods, it becomes progressively evident that the model is describing the underlying MOBC reasonably well. A similar pattern can be observed for number of drinks, norm violation, and commitment.

Cohort Results

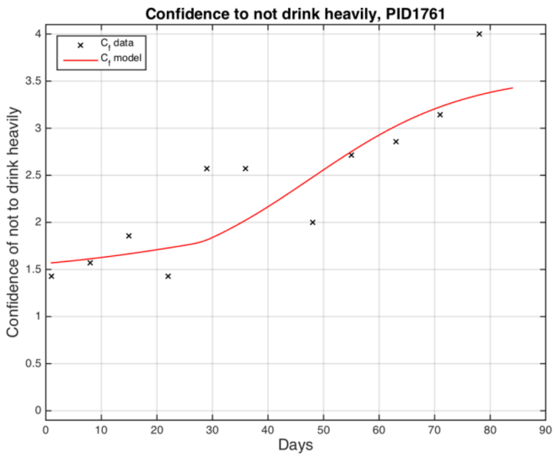
PID 1761. In Figure 4a, we can see that this patient reduces his drinking to a moderate level successfully, starting at a high level and reducing to an average of 1-2 drinks by the end of the treatment period, which is captured by the model solution (solid red line). The data show that there is a significant behavior change occurring between days 20 and 30 in treatment. This behavior is represented by the model solution, which indicates after approximately day 30, 1761 starts drinking less than his personal norm (dashed red line), and remains below this level for the rest of the treatment period.



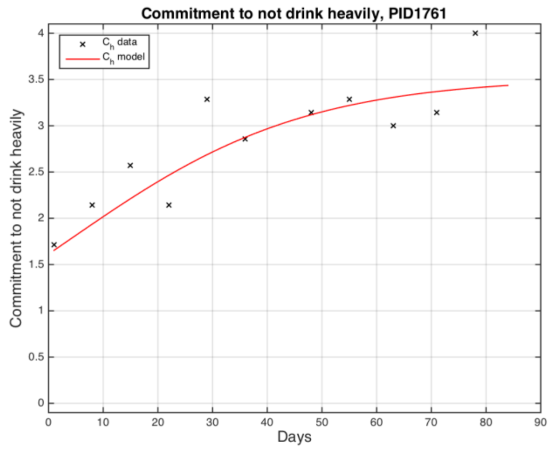
(a) Alcohol consumption



(b) Norm violation

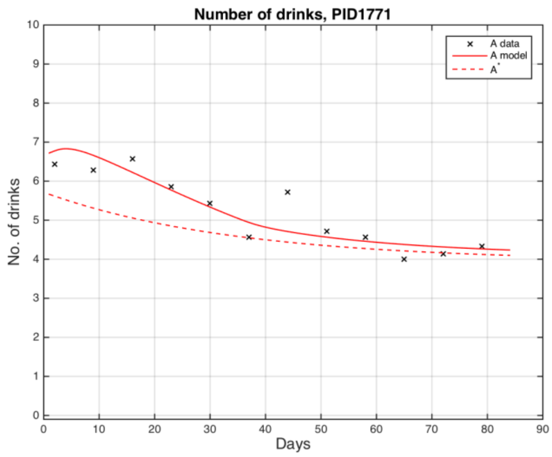


(c) Confidence

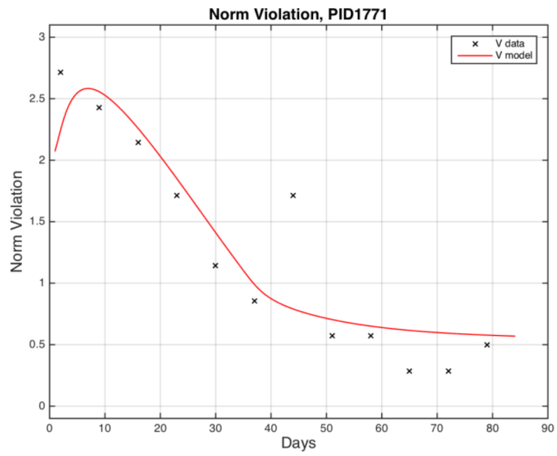


(d) Commitment

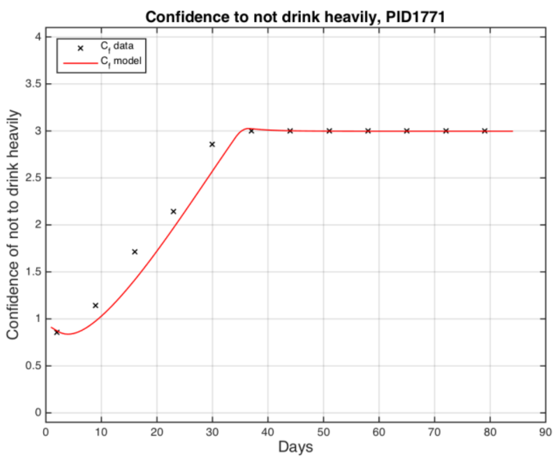
Figure 4. PID 1761 weekly averaged data and model solution. Estimated parameter values are a_1 = 0.548, a_2 = 0.286, a_3 = 0.035, a_4 = 0.043, v_1 = 0.103, v_2 = 0.085, d_1 = 0.014, d_2 = 0.114, b = 6.479, r = 0.036, l = 2.063, m = 0.046, k = 3.520, A_0 = 12.336, V_0 = 2.159, C_{f_0} = 1.570, C_{h_0} = 1.654, α = 1.342, and n = 3.583, with statistical error model weights $\boldsymbol{\gamma}$ = [0.4, 0.3, 0, 0].



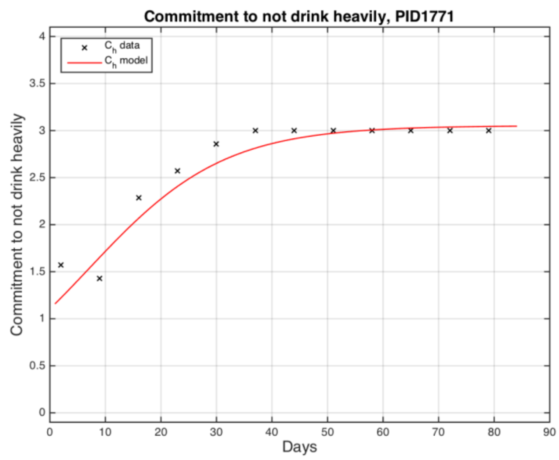
(a) Alcohol consumption



(b) Norm violation

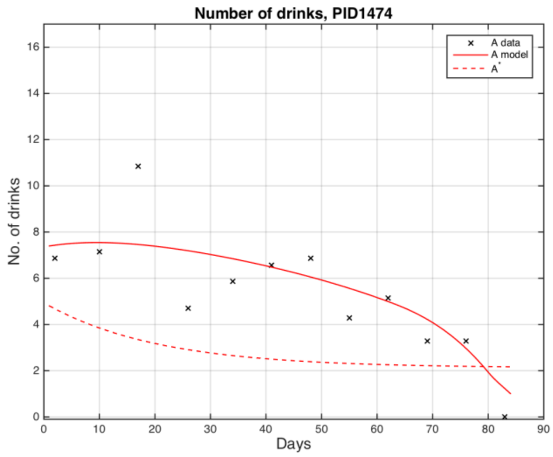


(c) Confidence

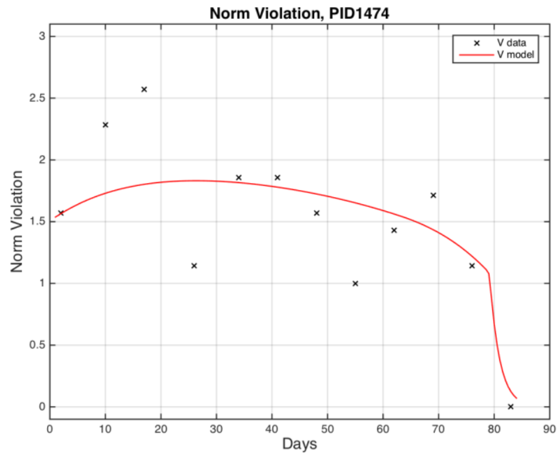


(d) Commitment

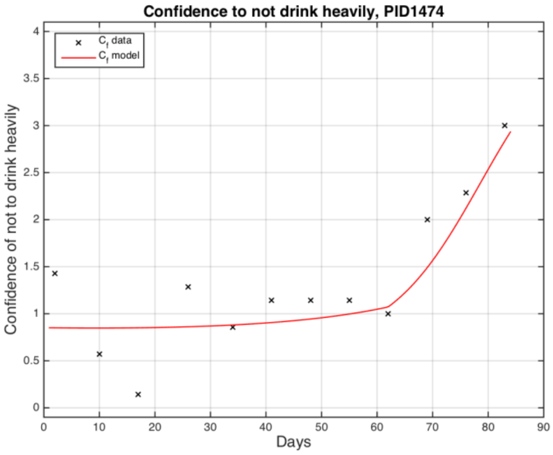
Figure 5. PID 1771 weekly averaged data and model solution. Estimated parameter values are a_1 = 0.465, a_2 = 0.155, a_3 = 0.053, a_4 = 0.024, v_1 = 1.638, v_2 = 0.141, d_1 = 0.556, d_2 = 0.245, b = 1.769, r = 0.029, l = 3.946, m = 0.082, k = 3.052, A_0 = 6.721, V_0 = 2.075, C_{f_0} = 0.909, C_{h_0} = 1.162, α = 0.122, and n = 2.997, with statistical error model weights $\boldsymbol{\gamma}$ = [0, 0.2, 0, 0].



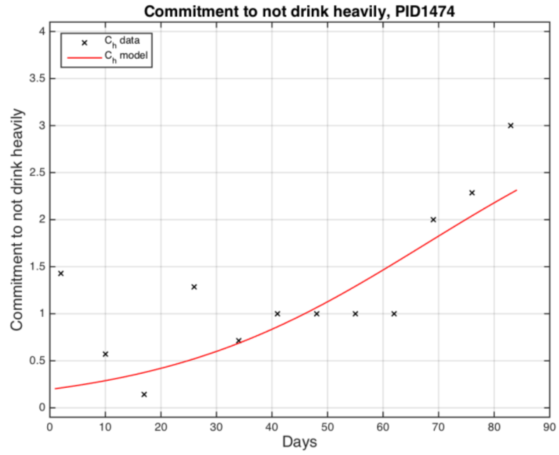
(a) Alcohol consumption



(b) Norm violation

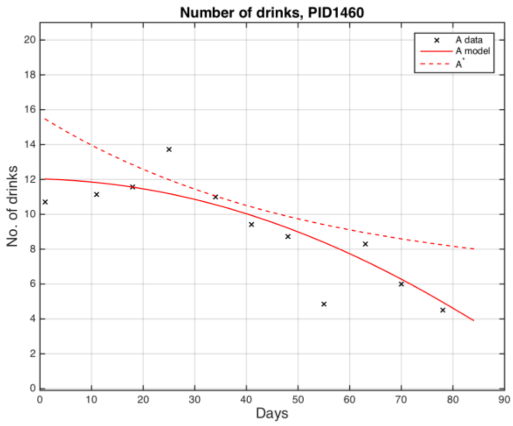


(c) Confidence

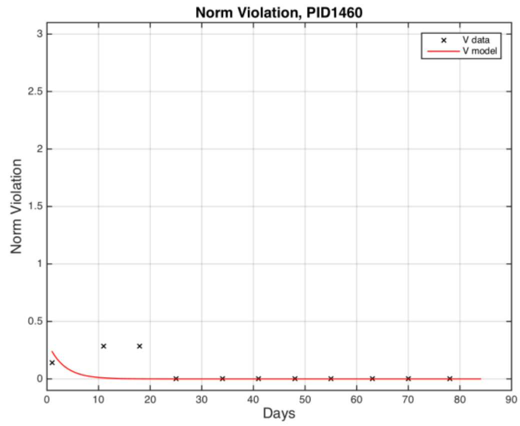


(d) Commitment

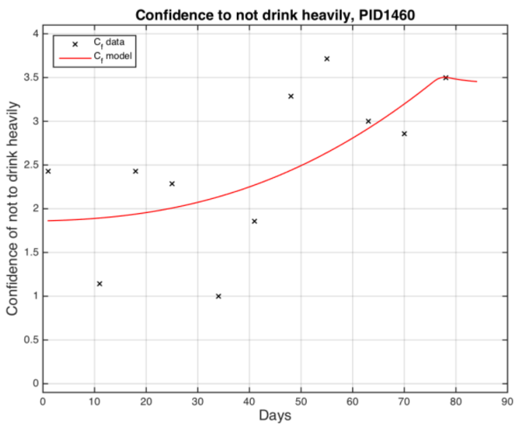
Figure 6. PID 1474 weekly averaged data and model solution. Estimated parameter values are a_1 = 0.306, a_2 = 0.168, a_3 = 0.005, a_4 = 0.077, v_1 = 0.174, v_2 = 0.561, d_1 = 0.094, d_2 = 0.144, b = 2.818, r = 0.049, l = 2.123, m = 0.042, k = 3.483, A_0 = 7.398, V_0 = 1.537, C_{f_0} = 0.851, C_{h_0} = 0.204, α = 0.661, and n = 4.108, with statistical error model weights $\boldsymbol{\gamma}$ = [0.2, 0, 0, 0].



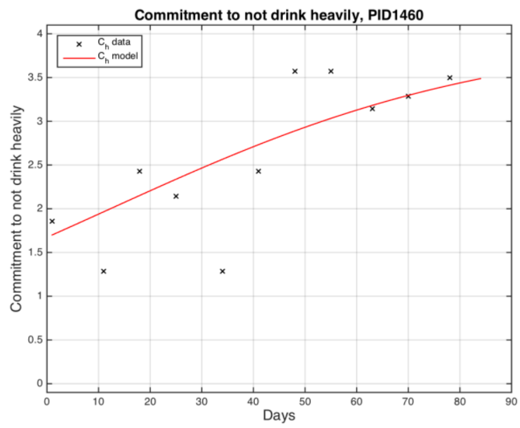
(a) Alcohol consumption



(b) Norm violation

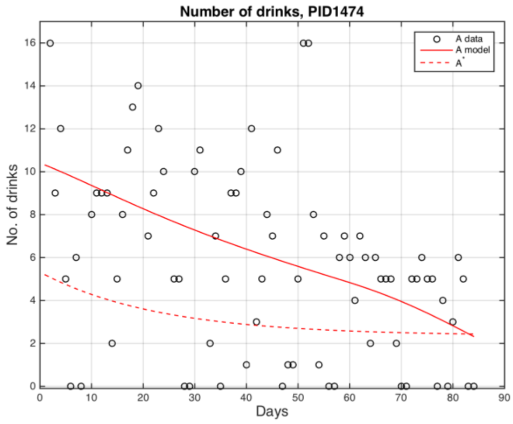


(c) Confidence

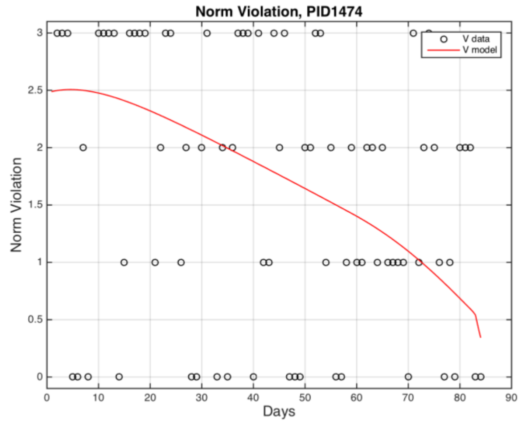


(d) Commitment

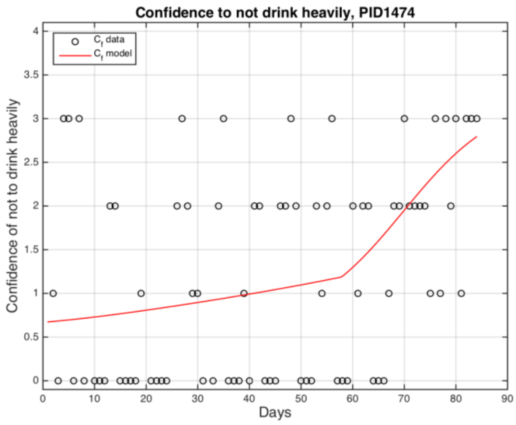
Figure 7. PID 1460 weekly averaged data and model solution. Estimated parameter values are a_1 = 0.120, a_2 = 0.000, a_3 = 0.068, a_4 = 0.005, v_1 = 0.270, v_2 = 0.325, d_1 = 0.083, d_2 = 0.224, b = 9.443, r = 0.020, l = 6.212, m = 0.027, k = 3.998, A_0 = 12.025, V_0 = 0.238, C_{f_0} = 1.863, C_{h_0} = 1.700, α = 0.764, and n = 3.428, with statistical error model weights $\boldsymbol{\gamma}$ = [0, 0, 0, 0].



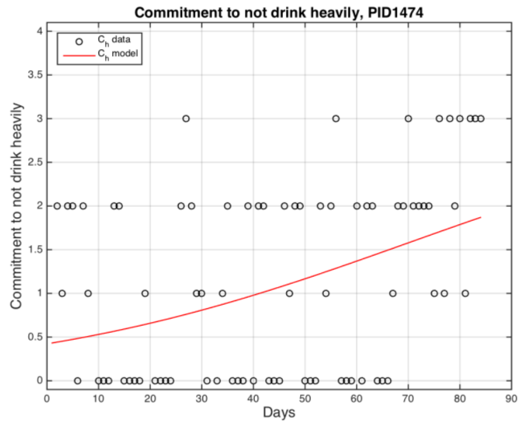
(a) Alcohol consumption



(b) Norm violation

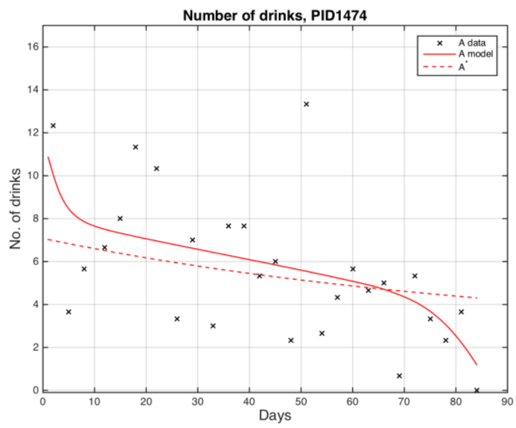


(c) Confidence

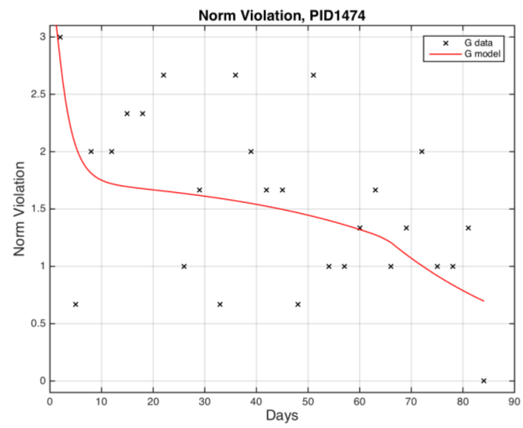


(d) Commitment

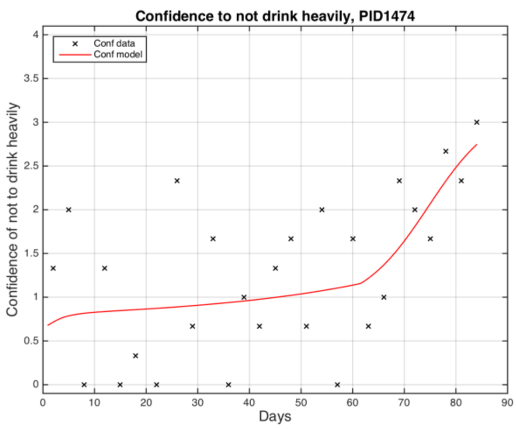
Figure 8. PID 1474 data and model solution. Estimated parameter values are a_1 = 0.336, a_2 = 0.154, a_3 = 0.092, a_4 = 0.041, v_1 = 0.381, v_2 = 0.472, d_1 = 0.124, d_2 = 0.134, b = 2.966, r = 0.043, l = 2.357, m = 0.027, k = 3.154, A_0 = 10.312, V_0 = 2.490, C_{f_0} = 0.674, C_{h_0} = 0.432, α = 0.620, and n = 3.263, with statistical error model weights $\boldsymbol{\gamma}$ = [0.5, 0, 0.3, 0].



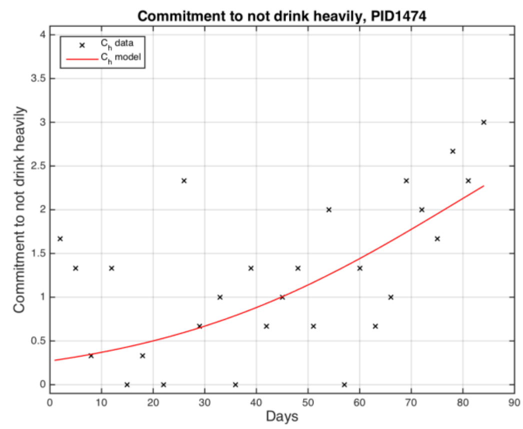
(a) Alcohol consumption



(b) Norm violation

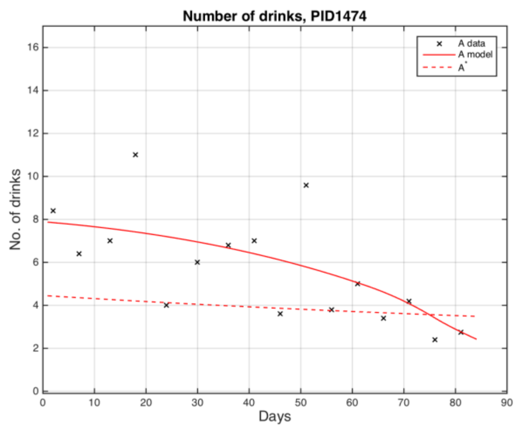


(c) Confidence

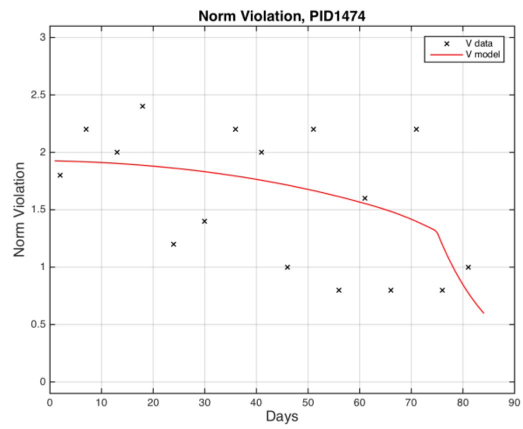


(c) Confidence

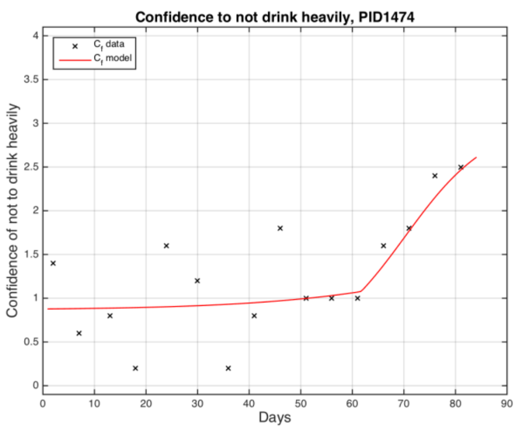
Figure 9. PID 1474 data averaged every 3 days and model solution. Estimated parameter values are a_1 = 0.898, a_2 = 0.527, a_3 = 0.013, a_4 = 0.145, v_1 = 0.517, v_2 = 0.031, d_1 = 0.154, d_2 = 0.212, b = 4.629, r = 0.011, l = 2.450, m = 0.034, k = 4.221, A_0 = 10.868, V_0 = 3.194, C_{f_0} = 0.680, C_{h_0} = 0.280, α = 0.884, and n = 3.188, with statistical error model weights $\boldsymbol{\gamma}$ = [0, 0, 0.2, 0.2].



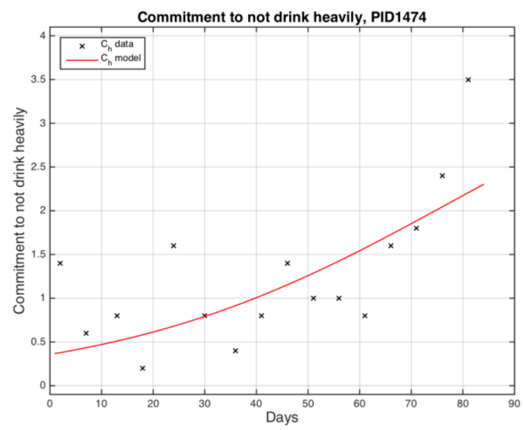
(a) Alcohol consumption



(b) Norm violation



(c) Confidence



(c) Confidence

Figure 10. PID 1474 data averaged every 5 days and model solution. Estimated parameter values are a_1 = 0.300, a_2 = 0.150, a_3 = 0.064, a_4 = 0.029, v_1 = 0.179, v_2 = 0.086, d_1 = 0.070, d_2 = 0.140, b = 2.321, r = 0.006, l = 2.137, m = 0.031, k = 4.236, A_0 = 7.871, V_0 = 1.926, C_{f_0} = 0.878, C_{h_0} = 0.368, α = 0.389, and n = 3.022, with statistical error model weights $\boldsymbol{\gamma}$ = [0, 0, 0, 0].

In Figure 4b, 1761's norm violation data is often above an average value of 2 (*Probably*) in the first month and then decreases quickly towards 0 (*Definitely Not*) for the remainder of the treatment period. This behavior is captured by our model solution.

In Figures 4c and 4d, the data and model solution show that there is an increase in both confidence and commitment as the patient decreases his drinking level. The patient starts the treatment period with a confidence and commitment level of approximately 1.5 (*Somewhat - Moderately)* and then increases towards the maximum level of 4 (*Extremely)*. Notice that in Figure 4c, the patient's confidence initially increases slowly until around day 30, at which point he stops drinking heavily. His confidence then increases rapidly after he has mastered the habit of drinking moderately.

PID 1771. In Figure 5a, we can see that the patient successfully reduced his drinking from a heavy to a more moderate level (average 6.5 to 4.2 drinks per day). The model solution in Figure 5a expresses the overall reduction in number of drinks during the treatment period. Although the patient's alcohol consumption decreases towards his personal norm, he never achieves this threshold. Note that the patient returns to drinking heavily around day 45. However, around this time the patient's confidence and commitment remain at his highest level, indicating that some other factor causes this high drinking. Thus, our model solution does not reflect this.

In Figure 5b, the norm violation data and model solution decrease from a high level (*Probably – Definitely)* to a low level *(Definitely Not – Possibly)* over the treatment period. The patient does not ever reach an averaged value of 0 (*Definitely Not*), but remains around 0.5 towards the end of the treatment period. This is indicated by the fact that his alcohol consumption stays above his personal norm in Figure 5a. We note that the patient has a higher norm violation around day 45 due to the heavy drinking around the same time.

In Figures 5c and 5d, the patient's confidence and commitment increase from approximately a level of 1 (*Somewhat*) to a level of 3 (*Very*) within the first month, and then remain at this level for the rest of the treatment period, as represented by both the data and model solutions.

Overall, this patient stops drinking heavily about a month into treatment. After this point, all four variables remain somewhat constant, indicating that he is most likely satisfied with his drinking habit (*Not Drinking Heavily).*

PID 1474. In Figure 6a we can see that, even though the data is a little sporadic, the trend of the patient's alcohol consumption decreases from heavy drinking to below an average of 4 drinks per day towards the end of the treatment period. The model solution follows a similar pattern. It also indicates that the patient reaches his personal norm around day 80, which is reasonable because his norm violation goes to an average value of 0 around the same day (Figure 6b).

Again, although the patient's norm violation data is a bit scattered (Figure 6b), overall we see a decrease over the treatment period. This decrease in norm violation is significant around day 80, which the model solution also agrees with.

In Figure 6c, the data shows that the patient's confidence remains low until around day 65, at which point it increases to 3 (Very Confident). This is reflected in the model solution, as confidence starts to increase immediately after the patient stops drinking heavily around day 65. Similarly, the model solution for commitment follows an overall increasing trend in commitment data (Figure 6d).

PID 1460. In Figure 7a, the patient starts the treatment period drinking heavily and then reduces his drinking on average to just below the heavy drinking threshold. These dynamics are well captured by the model solution. We note that this patient remains below his personal norm over the course of the treatment period, which explains why his norm violation data and solution decrease quickly to zero and remain there (Figure 7b). This suggests that norm violation is not as significant as confidence and commitment in reducing the patient's alcohol consumption.

In Figures 7c and 7d, we can see that although the confidence and commitment data are dispersed, the model solutions are able to exhibit the general increasing trend.

Discussion

This study used mathematical modeling as a complementary method to standard statistical approaches to help understand the dynamic process of behavior change in the context of alcohol use disorder. It demonstrates how mathematical modeling can be a tool to examine mechanisms underlying drinking reduction with a focus at the individual level, and in doing so, nuanced relationships between variables can be identified that might not have otherwise been determined through traditional statistical methods. While statistical methods are often used to determine factors that can explain successful and unsuccessful outcomes, the modeling effort here focuses on understanding how factors interact over time to produce the outcome. By building upon the work by Banks et al. (2016), this study extended the iterative effort of improving the original model by applying the model to three additional "treatment responders" - individuals who dramatically reduced their drinking during the study period and had more complete data. We fit the mathematical model to each patient's data to determine whether a common set of mechanisms emerged, such that the decrease in their alcohol consumption was explained by norm violation, confidence, and commitment. Through this application, and making adjustments to the model to better reflect the patterns of the data, a honed equation for behavior change emerged. Thus, we demonstrated the ability to iteratively move from a single-case model to a cohort with similar underlying MOBC. Next steps will include testing the model with other treatment responders that have less complete data to see if the model continues to hold across participants.

Some interesting findings result from this model building process. Alcohol consumption, norm violation, confidence and commitment in the model are allowed to increase or decrease at varying speeds allowing for each individual to demonstrate a unique speed and process of change. Furthermore, unlike previous traditional statistical work we have performed (Morgenstern et al., 2016), this model identified an important combined effect of commitment and confidence - confidence can be high, but without commitment, drinking does not decrease. In addition, while norm violation has been a construct of focus in studies on personalized feedback (Carey et al., 2010; Larimer et al., 2009), it has been less of a focus in the context of ongoing treatment. In collecting and evaluating daily data on whether a person evaluated their drinking as excessive, we identified a latent dynamic construct - one's personal definition of normative drinking - as being particularly important in influencing potential successful reduction in drinking in a more intensive treatment protocol, beyond feedback about drinking. Our modeling efforts suggest collecting information about a person's personal norm threshold would be an important area of future research.

Study Limitations

Given the developmental nature of this work, there are several study limitations to consider. Even though recent studies (Kuerbis et al., 2014; Morgenstern et al., 2016) were utilized to help us understand how the key variables interact with each other over time, the modeling process is slow due to the lack of previous work considering inter- and intra-personal factors relating to behavior change in patients with AUD that include non-linear relationships. Since this preliminary model is an initial step of our model building process, variables such as desire were held as constant temporarily. Indeed, in the SMART study, desire does not vary greatly throughout the treatment period. For the sake of simplicity, we decided to hold desire as constant in the initial step. Our next step will be to build our model by including a mathematical term for desire that more accurately and thoroughly fits the data. In addition, this study employed a secondary data analysis design where data were not collected for modeling purposes. Thus, the inability to identify a strong linear trend using daily data may reflect limitations in the data collection. For example, confidence, commitment, and norm violation were measured as discrete ordinal variables, whereas they are generally modeled as continuous variables since a patient probably feels a continuous change instead of a sudden jump from one level to another. New data is currently being collected that includes more response options to improve the quality of data in preparation for a next round of modeling. Furthermore, we utilized visual inspection and analysis of alcohol consumption to determine which participants to include in the iterative model development process, which inherently impacts the model results. This method also does not allow for generalizability to a larger group until the model has been tested for fit across a larger group of participants. The next step in our research will be to see if the model successfully applies to a wider group of problem drinkers who respond to treatment and have less complete data. We also intend to apply our model to unsuccessful patients to investigate whether the hypothesis of the *interactions* among the factors can shed light into why some patients respond to treatment and others do not. Given the sample used in this study, potential mechanisms identified here can only initially be considered to apply to problem drinking MSM rather than a wider population of problem drinkers.

Conclusion

Increasingly, behavior change is being seen as a complex, dynamic phenomena that operates at an individual level (Riley et al., 2011). For example, social learning theories that underlie most AUD behavioral interventions posit the individual level therapy outcomes are the results of interactions between traits, dynamic internal factors, contexts, treatments, and time. The nature of these interactions including the time frames for how variables (slow-moving versus fast acting) effect these interactions is as yet unknown. Attempting to use methods (e.g., modeling on the interpersonal or population level) that aggregate across individuals likely serves to obscure rather than clarify the nature of these interactions. Standard linear approaches, including multi-level modeling, are limited in handling complex interactions, such as nonlinear relationships and feedback loops, and especially those involving time (Tan et al, 2012). Mathematical modeling provides a useful complementary and supplementary approach to these standard methods as a way of identifying nuanced relationships between variables and for providing more information about future areas of exploration for MOBC research, including data collection procedures, new constructs of focus, and nonlinear relationships.

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Author Contributions

Modified the original model and conceived the method to average the data: HB, KB, and RE. Performed the computational work: KB, RE, SS, and LS. Wrote the first draft of the manuscript: SS. Contributed to writing of manuscript: RE, KB, and AK. Made critical revisions and approved final version: AK, HB, and JM. All authors reviewed and approved of the final manuscript. Co-investigators of Project SMART: JM and AK.

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