A simple qualitative model for the vibration of the vocal folds

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The purpose of this short note is to establish a simple qualitative model for the vibration of the vocal folds. Its basic assumptions are physically credible, its main aim is pedagogical and it can be explained with a minimum of mathematics. It is no real substitute for the more ambitious models of Flanagan and Landgraf (1975) and Tietze (1973) which take into consideration that the closure of the vocal folds is not uniform. The lower part closes first. Therefore these models represent the vocal folds by two masses on each side. Our model just uses one mass. Its assumptions are as follows.

A. The tensions and the masses of the muscles involved, mainly the vocalis and the cricothyroid muscles, determine a damped vibratory system S with a certain frequency $w/2\pi$ Hz.

B. The duration of the closed phase is half of the period of S.

C. The transglottal pressure P and a rebound B from S initiate the opening movement of the system with a certain force Q of very short duration.

D. When the movement initiated by Q has reached its maximum, a Bernoulli force R sets in which decreases proportionally to the opening between the vocal cords.

The assumption A is an uncontroversial simplification. The basic frequency w is determined by the tension T of S and its mass M in such a way that it decreases as T decreases and M increases. (The theoretical formula in the undamped case is $w^2=T/M$). The assumption B is motivated mainly by the observation that the closed and open phases of the glottal cycle are approximately equal. Since the modifications of the frequency w introduced by the Bernoulli force are never very large (see below), it has seemed natural to tie the duration of the closed phase to the state of the system S.

The assumptions C and D express a convenient way of circumventing the complicated interaction between the movements of the vocal cords and the flow through the glottis. D expresses the conventional view that the Bernoulli sucking force helps close the glottis but adds to it the assumption that the force sets in when the opening is maximal. This is natural because the Bernoulli force is largest when the flow is stationary and the natural moment for this to happen is when the opening of the glottis is maximal. As the glottis closes, the flow becomes less and less stationary. The assumption that the decrease of R follows the size of the opening is somewhat ad hoc but not unreasonable.

Note that the assumption C considers the force Q to be composed of subglottal pressure P and a rebound B. Under stable phonation, Q is of course constant. For a beginning phonation, the rebound vanishes for the first cycle but picks up to a stable value later. This fits with the increasing amplitudes of a beginning phonation.

The model seems to be able to explain a number of known facts about the vibration of the vocal folds.

Figure 1 shows the changing shape of the glottal cycle when S,P,Q are fixed and the Bernoulli force R increases /1/. When R is zero, the assumption A says that the open part of the glottis cycle follows a damped sine curve whose maximum is proportional to Q/M. The sine curve in the figure is only slightly damped. We see that the closing of glottis becomes faster as R increases. In this way, the curve representing the open phase acquires its characteristic asymmetric form (see e.g. Fant 1979 and Anathapadmanabha and Fant 1982) at the same time as the period of the glottal cycle decreases, i.e. as the frequency increases.

An increased Bernoulli force can be thought of as an efficient way of producing sound. Favourable conditions for this are regular movements of the vocal cords with no asymmetries or irregularities. It is probable that trained singers realize these conditions. A certain support for this statement is the fact that the ratio of the duration of the open to the closed phase is smaller for trained singers than for untrained ones (Sundberg and Gauffin 1979).

It can be shown mathematically (see the formula (3) of the appendix) that if S is fixed and Q and R are proportional to transglottal pressure P, then the duration of the glottal cycle is independent of P (Fig.2). But this is only approximately true. As a matter of fact, experiments have shown that when transglottal pressure increases, the duration of the glottal cycle decreases so that the frequncy increases. A reasonable explanation for this second order effect is that the opening force Q is not proportional to P for large values but subproportional, i.e. it increases less than proportionally to P. Since the maximal glottis opening is proportional to Q, this means that it grows less than proportionally to P. Then the formula (3) of the appendix shows that this has the same effect as an increase of the Bernoulli force R. Hence the duration of the glottis cycle decreases a little when P increases (Fig. 1).

Another thing that can be shown mathematically in the model is that the effect of the Bernoulli force R decreases when w increases as a result of increased tension and/or decreased mass of the system S. This means that if w is large, there is less asymmetry in the glottis curve, an effect which has been observed in a striking way by the sine-like glottograms of falsetto voice (Sundberg 1980 p. 64) /2/.

Mathematical appendix.

According to A, the glottal opening x as a function of t, x=x(t) satisfies the equation

$$Mx''(t) + CMx'(t) + Tx(t) = 0$$

during the open phase. The second term on the left accounts for the damping. For simplicity in this account, we put C=0. This restricts our formulas to the undamped or slightly damped case, but no new effects will appear unless the damping is very large.

Shortly after the impact of transglottal pressure, the function x(t) has the form

(1) $$x(t) = (Q/Mw)\ \sin wt\ ,\ w^2 = T/M,$$

which means that $Mx''(t) + Tx(t) = Q\delta(t)$ (to approximate the short-lived force Q by an instantaneous one seems legitimate). The function x(t) reaches its maximum Q/M when $t=\tau= \pi/2w$. After this, the equation of movement changes to

$$Mx''(t) + Tx(t) + (RM/Q)x(t) = 0.$$

Hence, the equation of the closing phase of the open part of the glottal cycle is

(2) $$x(t) = (Q/M)\ \cos W(t-\tau)$$

with t between τ and $\tau+ \pi/2W$ where W is given by

(3) $$W^2 = w^2 + (R/Q).$$

The formula (3) shows how an increased Bernoulli force makes the closure of the glottal cycle steeper and also that the quotient W/w decreases as w increases and R/Q is fixed. It also shows that if R and Q are proportional to P, then the period length $(\pi/w)+(\pi/W)$ is independent of P but that if Q increases less than a constant times P, then R/Q increases with P so that W also increases and hence the period length decreases.

The computations above can only be taken in a qualitative sense, although the model contains enough parameters to permit close fits to very regular observed glottis cycles. In my view, the importance of the model lies in the fact that the parameters have a very direct physical significance and that the model seems to permit direct and meaningful interpretations of many aspects of the glottal cycle.

/1/. The steep beginning of the curve is an artefact due to the assumption that the force Q is instantaneous.

/2/. In falsetto voice, the vocal folds do not close. The mathematical appendix covers this case too, if applied only to the open phases above a mean opening.

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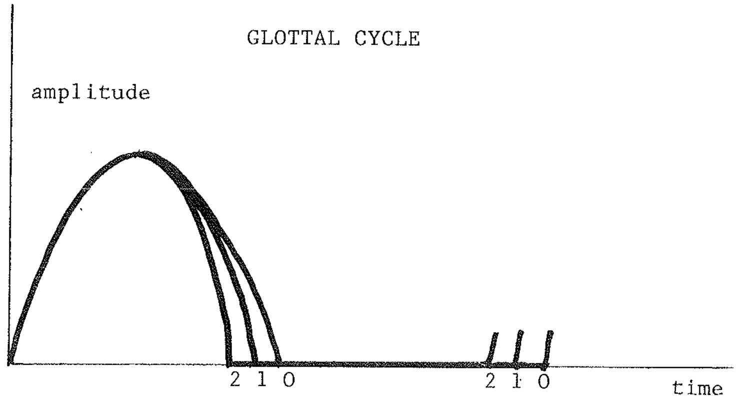

Fig. 1 Asymmetry and shortening of the length of the glottal pulse when the Bernoulli force R increases. The length of the closed phase is constant, curves with the same number correspond.

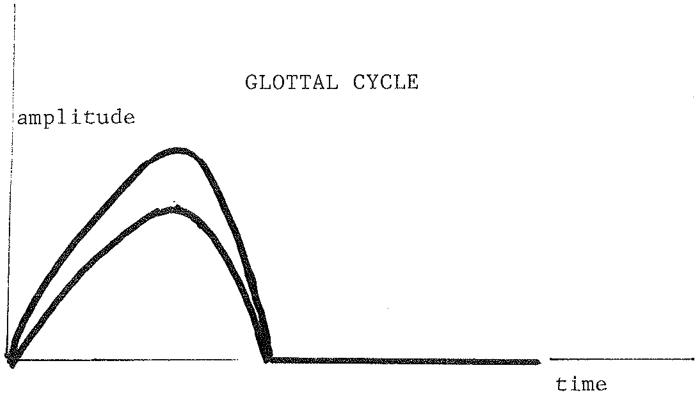

Fig. 2 The length of the glottal pulse is constant when the subglottal pressure P varies and the initial force Q and the Bernoulli force R are proportional to P